DIAGNOSTIC ASSESSMENT

CS 198-087: INTRODUCTION TO MATHEMATICAL THINKING
UC BERKELEY EECS
FALL 2018

The purpose of this assessment is to determine whether or not you will get value out of this course. If you can already attempt or answer at least half of these problems, this course may not be for you. One thing to keep in mind — these problems all have relatively short solutions. If you have any questions, don't hesitate to email us at **imt-decal@berkeley.edu**.

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1. What is the last digit of $3^{27} + 4^{27} + 7^{27}$?
2. How many factors does 2400 have? How many factors does it have that are multiples of 24?
3. Prove, using induction, that $(\sum_{i=1}^{n} i)^2 = \sum_{i=1}^{n} i^3$ for $n \in \mathbb{Z}^+$.
4. Find the sum of the coefficients in the expansion of $(xy + 1)^5$.
5. What is the sum of the roots of the polynomial $p(x) = x^{10} - 10x^9 + 7x^2 - 11x + 19$?
6. Prove Pascal's identity; that is, show that $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$.
7. Suppose I have a family of 7 adults and 5 children, and I can fit 6 other people in my car. How many ways can I pick the 6 people to come in my car, if I have to have at least 2 children come with me?
8. Consider the function $f(x) = x^2$. Give a domain and codomain such that $f(x)$ is
   a. a surjection, but not an injection
   b. a bijection
9. Determine the number of subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ that are not subsets of $\{2, 4\}$.
10. Suppose $A = \{t^2 : t \in \mathbb{N}_0, t \text{ is prime}, t < 5\}$, $B = \{2s : s \in \mathbb{N}_0, s < 10\}$ and $U = \{0, 1, 2, ...20\}$. Determine the following numbers or sets (where A^C represents the complement of set A):
    a. $|A \cap B|$
    b. $(A \cup B)^C$
    c. $(A^C \cap B)^C$