CS 198-087 Introduction to Mathematical Thinking
Fall 2018 Final

This is the final examination for Introduction to Mathematical Thinking.

- This exam has 8 questions (including question 0). The exam is out of 64 points.
- The exam will last for exactly 1 hour and 20 minutes, unless you have pre-arranged DSP accomodations.
- Fit all of your answers in the space provided.
- You are allowed to consult two double-sided, hand-written cheat sheets, but nothing else. No electronics.

DO NOT TURN THE PAGE UNTIL INSTRUCTED.

In the meantime, fill out the information on this page.

Name:

@berkeley.edu email:

Student ID Number:

Name of student to your left:

Name of student to your right:

0 Preliminary Questions

Points: 2 (1 each)

a) On a scale of 1 to 10, how are you feeling about this exam?

b) What was your favorite topic covered in this course?

1 ZOBOOMAFOO

Points: 14 (2/4/4/4)

a) Determine the number of permutations of ZOBOOMAFOO.

b) Determine the number of permutations of ZOBOOMAFOO, where ”ZBMA” appear next to each other, in any order. (e.g. ”ZAMB”, ”BMAZ” appear as substrings)

c) Determine the number of permutations of ZOBOOMAFOO, where the letters Z, B, M, A, F appear in alphabetical order. *(Hint: How can you model this using stars and bars?)*

d) Determine the number of three-letter strings made up of characters from ZOBOOMAFOO. (e.g. ”ZOO”, ”MBF”, ”OOO”, ”ZOF”)

2 Combinatorial Proofs

Points: 8

(This problem was modified after the start of the exam, this document reflects the updated version of these problems.)

Give a combinatorial proof of the following statement:

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$$

3 Primality

Points: 8

Prove that if p is a prime, $p \geq 5$, then $p = 6k+1$ or $p = 6k-1$ for some $k \in \mathbb{N}$.

4 Modular Arithmetic, Mechanical

Points: 8 (4/4)

a) Evaluate $15^{26} \pmod{23}$.

b) Determine $17^{-1} \pmod{63}$.

5 Fun...ctions

Points: 8 (3/5)

Suppose $f_k(x) = (x-1)(x-2)....(x-k)$. Notice that $f_k(x)$ is a polynomial of degree k.

a) What is the coefficient on x^{k-1}? (Your answer should be a function of k.)

b) What is the coefficient on x? (Your answer should be a function of k. You can leave it as a sum.)

6 Poly No Meal

Points: 8 (2/4/2)

Let $f(x) = (x^5 - 2x^{-3})^{12}$. Determine each of the following.

a) The sum of the coefficients in the expansion of $f(x)$

b) The general term t_k in the expansion of $f(x)$

c) The coefficient on x^{20} in the expansion of $f(x)$

7 Polynomial Interpolation

Points: 8 (4/2/2)

Suppose we want to find the polynomial that interpolates $\{(1,5),(2,6),(4,1)\}$ using Lagrange Interpolation.

a) Find $p_1(x)$, the sub-polynomial corresponding to $x_1 = 1$.

b) Now, suppose we want to find the interpolating polynomial under mod q, for some q. Why cannot we do this when $q = 12$? Give a concrete example of a calculation that cannot be done in mod 12.

c) For some q, the interpolating polynomial is $p(x) \equiv x + 4 \pmod{q}$. Determine q. Justify your answer.