EXTRA CREDIT ASSIGNMENT

CS 198-087: INTRODUCTION TO MATHEMATICAL THINKING
UC BERKELEY EECS
SPRING 2019

This assignment is worth 12 points to your raw grade. Each problem is worth 2 points. You must show your work to receive any credit. *It is due on Gradescope on Saturday, May 11th, at 11:59PM.*

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1. Determine the last digit of $13^{27} + 14^{27} + 15^{27}$.
2. Solve the following expression for x:

$$5x \equiv 3 \pmod{33}$$

3. Consider $f(x) = 2|x| - 3$. Give a codomain and codomain such that $f(x)$ is
   a. An injection, but not a surjection
   b. A surjection, but not an injection
   c. A bijection
4. Using a truth table, prove or disprove the following equivalence relation:

$$\neg((P \vee Q) \wedge (\neg R)) \equiv (\neg P \wedge \neg Q) \vee R$$

5. Determine the number of non-negative integer solutions to

$$w + x + y + z$$

such that $w > 3, x > 4, y \geq 5, z \leq 2$

6. Give a combinatorial proof of the following:

$$\sum_{i=0}^{n} \left(\binom{n}{i} \sum_{j=0}^{n-i} \binom{n-i}{j} \right) = 3^n$$

(Hint: When we proved $\sum_{i=0}^{n} \binom{n}{i} = 2^n$ with a combinatorial proof, we said that 2^n represented the number of ways to distribute a set of n items into two categories.