EXTRA CREDIT ASSIGNMENT

CS 198-087: INTRODUCTION TO MATHEMATICAL THINKING
UC BERKELEY EECS
FALL 2018

This assignment is worth 15 points to your raw grade. Point values for each problem are given. You must show your work to receive any credit. *It is due on Gradescope on Friday, December 7th, at 11:59PM.*

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1. (1 pt) Determine the last digit of $13^{27} + 14^{27} + 15^{27}$.
2. (1 pt) Solve the following expression for x:

   $$5x \equiv 3 \pmod{33}$$

   (Hint: Look at the "Division in Modular Arithmetic" slide of Lecture 6.)
3. (1 pt) Consider $f(x) = 2|x| - 3$. Give a codomain and codomain such that $f(x)$ is
   a. An injection, but not a surjection
   b. A surjection, but not an injection
   c. A bijection
4. (1 pt) Using a truth table, prove or disprove the following equivalence relation:

   $$\neg((P \vee Q) \wedge (\neg R)) \equiv (\neg P \wedge \neg Q) \vee R$$

5. (1 pt) Determine the number of non-negative integer solutions to

   $$w + x + y + z$$

   such that $w > 3, x > 4, y \geq 5, z \leq 2$

6. (1 pt) Suppose

   $$f(x) = \sum_{k=0}^{5} (-1)^k \binom{5}{k} (5-k) x^{4-k} 3^k$$

   Determine $f(2)$. *(Hint: First, show that $f(x)$ is the derivative of some binomial expansion.)*
7. (2 pt) Suppose $t_n = a + (n-1)d$, where $n \in \mathbb{N}$ and $n, d \in \mathbb{R}$. Prove, using induction, that

   $$\sum_{i=1}^{n} t_n = \frac{2a + (n-1)d}{2} n$$

8. (2 pt) Given the triangle inequality, $|a + b| \leq |a| + |b|$, prove that

$$|a_1 + a_2 + ... + a_n| \leq |a_1| + |a_2| + ... + |a_n|$$

using induction.

9. (2 pt) Give a combinatorial proof of the following:

$$\sum_{i=0}^{n} \left(\binom{n}{i} \sum_{j=0}^{n-i} \binom{n-i}{j} \right) = 3^n$$

(Hint: When we proved $\sum_{i=0}^{n} \binom{n}{i} = 2^n$ with a combinatorial proof, we said that 2^n represented the number of ways to distribute a set of n items into two categories.

10. (3 pt) Suppose the polynomial $x^3 - \alpha x^2 + \beta$ has three roots, one of which is equal to β^2.
    a. What is the sum of all possible values of β?
    b. What is the product of all non-zero possible values of β?