Problem Set 2: Number Sets, Propositional Logic

CS 198-087: Introduction to Mathematical Thinking
UC Berkeley EECS
Fall 2018

This homework is due on Monday, September 17th, at 6:30PM, on Gradescope. As usual, this homework is graded on participation, but it is in your best interest to put full effort into it. This is a good opportunity to learn how to use LaTeX.

---

1. Determine a bijection $f : A \to B$ between each pair of sets, and prove that f is a bijection (i.e. show that it is both an injection and surjection).

   a. $A = \{1, 2, 3, 4, 5, 6, ...\}, B = \{4, 7, 10, 13, 16, 19, ...\}$

   b. $A = \{2, 4, 6, 8, 10, 12, ...\}, B = \{2, -2, 3, -3, 4, -4, ...\}$

2. Write the equivalent of the following statements using propositional logic.

   a. There exists an integer solution to the equation $x^2 + 2x + 1 = 0$.

   b. There are no three positive integers a, b, c that satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2.

   c. $\sqrt{2}$ is a rational number.

   d. For $|r| < 1, r \in \mathbb{R}$, the sum of the series $1 + r + r^2 + r^3 + \cdots$ is equal to $\dfrac{1}{1-r}$.

3. Determine the contrapositive and converse of each of the following statements.

   a. If it rains tomorrow, I will bring an umbrella.

   b. If the clock is between 12PM and 2PM and I am hungry, then it is lunch time.

   c. If your final grade in this course is at least 70%, you will pass.

   d. If two sets A, B are disjoint, then the cardinality of their intersection is 0.

4. Using truth tables, prove that the following equivalences hold:

   a. $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$

   b. $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$

   c. $P \implies Q \equiv \neg P \vee Q$

5. Suppose that $P(x)$ and $Q(x, y)$ are propositional statements. Find the negation of the following statements:

a. $\exists x P(x)$

b. $\forall x P(x)$

c. $(\forall x)(\exists y) Q(x, y)$

d. $(\exists x)(\forall y) Q(x, y)$

Hint: replace $P(x)$ with a real propositional statement, i.e. "x is even."

6. In general, you cannot reverse the order of different existential quantifiers. That is,

$$(\forall x)(\exists y) P(x, y) \not\equiv (\exists y)(\forall x) P(x, y)$$

Give two examples of propositions $P(x, y)$ that illustrate why this does not hold.