Lecture 23: Review

Introduction to Mathematical Thinking

April 25, 2019

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Announcements

- Quiz 5 on Tuesday
 - This week and next week still count for attendance
- Homework 8 due Sunday 11:59pm
- Final feedback form released: please fill it out!

Today: More review.

Next week:

- Tuesday: Quiz only, class will be done early
- Thursday: Wrap-up lecture. Will talk about course advice and look at a cool application of some of the stuff we've seen.

extra credit
released after
quit

$$Z_3 = \{0, 1, 2\}$$

Example: In the following, Z_n refers to the set of integers modulo n.

a) Is
$$f: x \mapsto 7x$$
 a bijection from Z_{12} to Z_{12} ?

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$$f:x\mapsto 7x$$
 a bijection from Z_{12} to Z_{12} ? $\gcd\left(\ 7,\ 12 \ \right)=1$ b) Is $f:x\mapsto 6x$ a bijection from Z_{12} to Z_{12} ? $\gcd\left(\ 6,\ 12 \ \right)\ne 1$

c) Does there exist an injection from Z_{12} to Z_{24} ? A surjection? In both cases, if so, identify them.

injection
$$f(a)=f(b)=a=b$$
 $7a=7b$
 $7^{-1}7a=7^{-1}7b$
 $a=6$

$$f(2) = 0$$

$$f(4) = 0$$

$$2 \neq 4$$

$$not inj$$

$$not bij.$$

surjection suppose C is the output of
$$f$$
. $f(x) = C =$ $7x = C$ $\chi = 7^{-1}C$

$$7x = C$$

$$\chi = 7^{-1}C$$
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f: x +> x |A| \(\text{IB} \) surjection
[A]2|B|
...
...
...
...

Example: Suppose A,B are two sets.

- a) Suppose |A|=|B|=n. How many bijections are there from A to B?
- b) Suppose |A|=a and |B|=b. How many functions are there from A to B?
- c) Suppose again that |A|=a and |B|=b, and a< b. How many injections are there from A to B?

a)
$$\begin{array}{ccc}
a_1 & b_1 \\
a_2 & b_3 \\
\vdots & \vdots \\
a_n & b_n
\end{array}$$

$$\begin{array}{ccc}
N \cdot (n-1) \cdot (n-2) \cdot & & \\
& = N \cdot & & \\
\end{array}$$

b)
$$x_{1}$$
 y_{1} y_{2} y_{2} y_{3} y_{4} y_{5} y_{6} y_{7} y_{8} y_{1} y_{2} y_{3} y_{4} y_{5} y_{7} y_{8} y

Example: Give a combinatorial proof of the following identity, assuming $n \geq r \geq k$:

$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$$
Choosing a team of r players r being starters

LS: I) choose the tearn $\binom{n}{r}\cdot\binom{r}{k}$

2) choose the states
$$\binom{n}{r}\cdot\binom{n-k}{r-k}$$

RS: I) choose the states $\binom{n}{k}\cdot\binom{n-k}{r-k}$

2) choose rest of

Example: Prove the power rule of derivatives, using the Binomial Theorem. That is, prove

Example: Prove the power rule of derivatives, using the Binomial Theorem. That is, prove
$$\lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$
for $n \ge 1$.

Hint: You will need the limit definition of derivatives.
$$(x+h)^n = \lim_{h \to 0} \frac{1}{h} + \lim_$$

Example: In the expansion of

$$(2x-rac{c}{x})^{10}$$

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the value of the constant term (i.e. the term independent of x) is 100000. Determine c.

$$t_{k} = \binom{10}{k} (2x)^{10-k} (-(x^{-1})^{k} (x+y)^{0} = \sum_{k=0}^{10} \binom{10}{k} x^{0-k} y^{0}$$

$$= (-1)^{k} \binom{10}{k} 2^{10-k} C^{k} (x+y)^{0} = \sum_{k=0}^{10} \binom{10}{k} x^{0-k} y^{0}$$

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$$= (-1)^{k} \binom{10}{k} 2^{10-k} C^$$

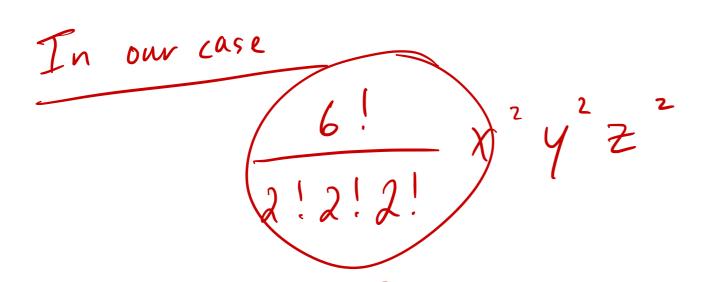
Example: What is the coefficient of the $x^2y^2z^2$ term in the expansion of $(x+y+z)^6$?

General?

$$\frac{n!}{a!h!c!} \chi^a \gamma^b z^a$$

$$a+b+c=n$$

$$0 \leq a,b,c \leq 7$$





Example: If the sum of the reciprocals of the roots of the quadratic

Example: Let
$$f(x) = x^4 - 14x^3 + 15x - 13$$
 have roots r_1, r_2, r_3 . Find
$$r_1^2 r_2 r_3 r_4 + r_1 r_2^2 r_3 r_4 + r_1 r_2 r_3^2 r_4 + r_1 r_2 r_3 r_4^2$$

$$= r_1 r_2 r_3 r_4 \left(r_1 + r_2 + r_3 + r_4 \right)$$

$$= -187 - 187 -$$