QUIZ 1

CS 198-087: INTRODUCTION TO MATHEMATICAL THINKING
UC BERKELEY EECS
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You will have 30 minutes to work on the quiz. Please fit all of your answers in the space provided. You are not allowed to consult any notes or use any electronics.

Total points: 7 + 9 + 9 = 25

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1. *True or False (7 pts, 1 pt each)*

   Circle either true or false in each of the below. No justification is needed.

   a. **True** or **False**: The contrapositive of the statement "if x is even, then x^2 is odd" is "if x^2 is even, then x is odd."

   b. **True** or **False**: The set of all positive multiples of 37 is uncountably infinite.

   c. **True** or **False**: The function $f(x) = x^4 - x^2$ with domain and codomain $\mathbb{R}$ is injective.

   d. **True** or **False**: For any two finite sets A and B, $|A - B| = |A| - |A \cap B|$.

   Consider sets $S = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$ and $T = \{1, 0, -1, \frac{1}{2}\}$.

   e. **True** or **False**: The mapping $w \mapsto \cos(w)$ is a bijection from S to T.

   f. **True** or **False**: There exists a bijection $f : S \to T$.

   g. **True** or **False**: Suppose $g : S \to T$ is a mapping. It is possible for g to be an injection, but not a surjection.

2. *Set Operations (9 pts, 3 pts each)*

   Consider the following sets, where the universe is $\mathbb{N}$:

   - $A = \{x : x \text{ is prime}\}$
   - $B = \{2k : k \in \mathbb{N}\}$
   - $C = \{x : (x \leq 30) \wedge (x = 6k, k \in \mathbb{N})\}$
   - $D = \{18, 24, 73, 4\}$

   Using **only** the above sets and any set operations (union, intersection, difference, Cartesian product, and complement), construct the following sets. For example, to create the set $\{73\}$, we can perform the operation $A \cap D$ or $D - B$. *(There may be more than one potential answer, but you only need to identify one.)*

   a. $\{18, 24, 4\}$

   b. $\emptyset$ *(the empty set)*

   c. $\{2, 6, 12, 30\}$

Of the following two questions, complete one.

3. *Fun at the Zoo (9 pts, 3 pts each)*

   Consider the statement, "if pigs can fly, then dogs can run or gorillas are not humans."

   a. Write this statement using propositional logic. Define any variables that you are using (e.g. P: "pigs can fly").

   b. Determine the contrapositive of this statement, written in English.

   c. Determine the negation of this statement, written in English.

4. *NAND Gates (9 pts, 3 pts each)*

   We define the NAND operation, $A \uparrow B$, to be false only when both A and B are true, and true in all other cases.

   a. Re-write $A \uparrow B$ using just $\vee, \wedge$, and $\neg$. Prove your result using a truth table.

   b. Using just the NAND operation ($\uparrow$), create an expression that is equivalent to $\neg A$.

   c. Using just the NAND operation ($\uparrow$), create an expression that is equivalent to $A \vee B$. *(Hint: Re-write your result from part a using De Morgan's Laws, then use your result from part b.)*