QUIZ 4

CS 198-087: INTRODUCTION TO MATHEMATICAL THINKING
UC BERKELEY EECS
SPRING 2019

You will have 30 minutes to work on the quiz. Please fit all of your answers in the space provided. You are not allowed to consult any notes or use any electronics.

There are **37** possible points on this quiz, but your score will be out of **27**, and is capped at 100% (e.g. even if you get all 37 points, your score is still 27/27). This allows you to attempt all problems, but doesn't penalize you if you are unable to finish one of them.

Name:

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1. *Counting Functions: 9 points (3/3/3)*

   Suppose A, B are two sets.

   a. Suppose $|A| = |B| = n$. How many bijections are there from A to B?

   b. Suppose $|A| = a$ and $|B| = b$. How many functions are there from A to B?

   c. Suppose $|A| = a$ and $|B| = b$, and $a < b$. How many injections are there from A to B?

2. *Permutations: 10 points (3/3/4)*

   Consider the string BIGBILLY.

   a. How many permutations are there of this string?

   b. How many permutations are there of this string, where ”BILL” appears as a substring?

   c. How many 3 letter strings are there, consisting solely of the characters in BIGBILLY? *(Hint: Consider two cases: One where all characters are unique, and one where two characters are repeated.)*

3. *Stars and Bars: 12 points (3/3/3/3)*

Suppose I have 100 $1 dollar bills that I want to distribute between three of my friends, LeBron, Lonzo and Lance.

How many ways can this be done...

a. In general, with no restrictions (other than that everyone receives some non-negative integer amount)?

b. If everyone receives at least $1?

c. If everyone receives at least $t, for $0 \leq t \leq 33$?

d. Such that LeBron and Lonzo receive the same amount? (*Hint: How can we format this as solving the number of solutions to* $x + y = 50$*?*)

4. *Combinatorial Proof: 6 points*

   Give a **combinatorial proof** of the following identity (assuming $n \geq r \geq k$):

$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$$

   (Note: Remember, combinatorial proofs are arguments that both the LHS and RHS count the same quantity. Algebraic proofs, i.e. ones that manipulate one side of this equation and show that it is equal to the other, will be given no credit.)